

# 线性代数 II (H) 2022 春夏期末

21 图灵 QJJ 回忆卷

2022 年 6 月 19 日

1. (20 分)  $M = \mathbb{Z}_{12} \oplus \mathbb{Z}_{18} \oplus \mathbb{Z}_{100} \oplus \mathbb{Z}^{30}$ ,  $N = \mathbb{Z}_2 \oplus \mathbb{Z}_x \oplus \mathbb{Z}_{900} \oplus \mathbb{Z}^y$ , if  $M$  and  $N$  are isomorphic in  $\mathbb{Z}$ -modules, find  $x$  and  $y$ .

2. (20 分) Find the similar canonical form of  $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}$  on  $\mathbb{Q}$ ,  $\mathbb{R}$  and  $\mathbb{C}$ .

3. (10 分) Show that  $y^3 - (x^7 + 3)^2y - 11x^7 - 33$  is irreducible on  $\mathbb{Q}[x, y]$ .

4. (10 分) If  $p, q$  are prime on PID  $R$ , and  $Rp \neq Rq$ , show that  $Rp^m + Rq^n = R$  where  $n, m \in \mathbb{Z}$ .

5. (10 分)  $I$  is an ideal of commutative ring  $R$  with identity, show that there is an ideal  $M$  satisfying  $I \subseteq M$  and  $R/M$  is a field.

6. (10 分) Show that there exist  $A, B \in M_3(\mathbb{Z})$ , such that  $AB = BA = E$  and the first row of  $A$  is  $(10, -12, 15)$ .

7. (20 分) Find the dimension of vector space  $\mathbb{Q}[x]/(x^3 + x) / (x^2 + 1)/(x^3 + x)$ .