Discrete Mathematics Quiz 2

2021 - 2022 春夏学期 郑文庭班

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1.

(a) Find the next larger permutation in lexicographic order after 76154238. (5%)

(b) Find the next larger 5-combinations of the set {1, 2, 3, 4, 5, 6, 7, 8} after {3, 4, 5, 7, 8}. (5%)

2. Allowing repeated characters, how many strings of six characters from {'0' ~ '9'}: (24%)

(a) contain '1'?

(b) contain exactly one '1'?

(c) contain '1' and '2' , where '1' is somewhere to the left of '2' in the string?

(d) contain '1' and '2' , where '1' is somewhere to the left of '2' in the string, with all the characters distinct?

(e) begin and end with the same character?

(f) consists of exactly 3 different characters?

3. Find the coefficient of x^9 in the expansion of $(x/2 - 3/x)^{15}$. (6%)

4. How many relations are there on a set with n elements that are: (24%)

(a) both reflexive and symmetric?

(b) both symmetric and antisymmetric?

(c) neither reflexive nor irreflexive?

(d) transitive? (if $n = 2$)

(e) equivalence relation? (if $n = 4$)

(f) partial order relation? (if $n = 2$)

(**Tips**: A relation R on the set A is **irreflexive** if for every $a \in A$, $(a, a) \notin R$.)

5. Write the first 6 terms of the sequence determined by the generating function: $(1 + x)/(1 - x)$. (10%)

6. Find the solution to the following iteration relation: (16%)

$$a_n = 5a_{n-1} - 6a_{n-2} + 2^n + 3 \ \textit{for}\ n \geq 2 \ \textit{where}\ a_0 = 1, \ a_1 = 1.$$

7. Let A be a set consisting 10 integers $10 \leq a_i \leq 99$ that are different from each other. Prove that A must have two different subsets without common elements, and the sums of the elements in these two subsets are equal. (10%)