Discrete Mathematics Quiz 1

2021 - 2022 春夏学期 郑文庭班

Shd0wash

2023 年 6 月 2 日

1. Determine whether the following statements are true of false. (12%)

(a) The following two propositions are logically equivalent: $p \to (\neg q \land r)$, $\neg p \lor \neg(r \to q)$.

(b) If A, B and C are sets, then $A - (B \cap C) = (A - B) \cup (A - C)$.

(c) This proposition is a tautology: $((p \to q) \land \neg p) \to \neg q$.

(d) $\mathrm{P}(A) = \mathrm{P}(B)$, if and only if $A = B$, where $\mathrm{P}(X)$ is the power set of X.

2. Suppose the varible x represents students, y represents courses, and $\mathrm{T}(x, y)$ means "x is taking y." Translate the statements into symbols. (16%)

(a) No course is being taken by all students.

(b) Every student is taking at least one course.

(c) Some courses are being taken by no students.

(d) No student is taking all courses.

3.

(a) Write a proposition equivalent to $p \lor \neg q$ that uses only p, q, $\neg$, and the connective $\land$. (4%)

(b) Write a proposition equivalent to $p \land q$ using only p, q, and the connective $|$. (8%)

("$|$" represents NAND. The proposition $p \mid q$ is true when either p or q, or both, are false; and it is false when both p and q are true)

4. There is a proposition formula with three varibles p, q, *and* r that is true when atmost one of the three varibles is true, and false otherwise.

(a) Express the proposition formula in full disjunctive normal form. (8%)

(b) Express the proposition formula in full conjunctive normal form. (8%)

5. Prove that the distribute law $A_1 \cap (A_2 \cup \ldots \cup A_n) = (A_1 \cap A_2) \cup \ldots \cup (A_1 \cap A_n)$ is true for all $n > 2$. (12%)

6. Adapt the Cantor diagonalization argument to show that the set of positive real numbers less than 1 with decimal representations consisting only of $0s$ and $1s$ is uncountable. (12%)

7. Find the "best" big-O notation to describe the complexity of the algorithm.

(a) An algorithm that prints all subsets of size three of the set $\{1,\ 2,\ 3,\ \ldots\ ,\ n\}$. (4%)

(b) The best-case analysis of a linear search of a list of size n (counting the number of comparisons). (4%)

8. Use mathematical induction to prove that $n^{n+1} > (n+1)^n$ for all positive integer $n \leq 3$. (12%)